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Homogeneous Turbulent Field within a Rotating Frame

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With a view to explaining the behavior of turbulent flows in turbomachine rotors, a turbulent shear flow is examined in a rotating coordinate system. When the turbulent field is homogeneous, equations of second order correlations are written at two points. In order to carry out spectral computations, triple correlations are neglected. Coriolis forces are found to have an effect on the stability of turbulence. Consequences concerning turbulence modeling and applications to turbomachinery are discussed.

Nomenclature

a_{ijkl} = integral function of Φ_{ij} and K_l
F,G = isotropic functions
K_i = wave vector
l = mixing length
l_0 = mixing length without rotation
n^1,n^2 = unit vectors of the local frame, normal to $\boldsymbol{K}$
n^3 = unit vector of the local frame, parallel to $\boldsymbol{K}$
N_1,N_2 = diagonal components of Φ_{ij} in the local frame
P = real part of the extradiagonal component of Φ_{ij} in the local frame
p = pressure fluctuation
Q = imaginary part of the extradiagonal component of Φ_{ij} in the local frame
$\overline{q^2}$ = turbulent kinetic energy
Ri = Richardson number
S = shear $\lambda_{ij}=S\delta_{i2}\delta_{j3}$
t = time
U_i = velocity
$\bar{U}_i$ = mean velocity at the point M
$\bar{U}_i'$ = mean velocity at the point M'
u_i = velocity fluctuation at the point M
u_i' = velocity fluctuation at the point M'
X_i = coordinates of M
X_i' = coordinates of M'
$Z_{ij}(|K|)$ = pressure velocity spectrum
α = angle between the principal directions of $\overline{u_iu_j}$ and the axes of the frame
β = constant
γ = angle between the principal directions of $\overline{u_iu_j}$ and of $\partial\bar{U}_i/\partial X_j+\partial\bar{U}_j/\partial X_i$
δ_{ij} = 1 when $i=j$, and 0 when $i\neq j$
Δ_{ij} = $\delta_{ij}-K_iK_j/K^2$
ϵ_{ijk} = Ricci's tensor
$\Phi_{ij}(\boldsymbol{K})$ = double correlation spectrum
$\Phi_{ij}^M(\boldsymbol{K})$ = modeled form of Φ_{ij}
$\varphi_{ij}(|K|)$ = integral of Φ_{ij}
λ_{ij} = mean gradient $=\partial\bar{U}_i/\partial X_j$
ν = fluid kinematic viscosity
Ω_i = rotation of the frame
ρ = density
$\Sigma_i(\boldsymbol{K})$ = pressure velocity spectrum
θ_{ijk} = triple correlation spectrum
ξ_i = vector MM'

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I. Introduction

IT is now well known that a rotating motion of the frame has strong effects on turbulent flows. The problem of predicting those effects is of interest mainly because of its applications to turbomachinery, for computing flows inside rotors by using rotating coordinate systems.[1] Furthermore, a better understanding of those effects is likely to provide enlightening information even about steady frame flows, in particular concerning the role played by the rotational part of mean flows such as shear flow.

Although some results, not very far from experiments, might have been obtained when computing rotating flows by using the same closure relations as in Galilean frames, the validity of those closures seems, in fact, to be very questionable in rotating frames, since turbulence is not materially indifferent. This has been pointed out by Lumley.[2] In particular, no reason can be put forward a priori to extend to rotating frames the classical modeling of the pressure strain correlation used to close one-point equations, or any other modeling involving isotropic functions.

In the present paper, we propose to exactly take into account, at least one part of the action of rotation, the linear and homogeneous contribution. The two-point correlation equations are therefore written and a linear study (rapid distortion) is made. The particular case of a shear flow uniform and steady in the rotating frame is studied. Obviously, such a linear study will not provide information about the nonlinear action of rotation which exists (Wigeland and Nagib[3]), but the value of our approach appears as soon as we consider that one of the most important consequences of rotation encountered in turbomachinery is due to a linear process: the coupling effect between Coriolis forces and production of turbulence by shear. In this case, experiments have shown that rotation has a strong effect on the stability of turbulence.[4,5] Either augmentation or suppression of turbulence intensities may result, and it is known that consequences concerning the behavior of flows inside centrifugal compressors are of prime importance for the engineers.[6]

Since the nonlinear transfer of energy between eddies of various sizes is not present in a rapid distortion model, we cannot pretend that our results will always represent the real behavior of flows such as encountered in turbomachines. The present computation will only provide valuable quantitative results concerning the evolution of turbulence during times which are short when compared to the characteristic times of the action of the turbulent motion on itself.

Concerning more general flows, in which nonlinear effects are important, our results are providing qualitative information. Furthermore, they may be helpful to develop accurate models of the linear part of the pressure terms which will be used to predict rotating flows with methods, taking into account triple correlations, for example: one-point equation methods. For this purpose, the influence of the rotating motion of the system on the accuracy of the closure relations currently used in Galilean frame is examined in Sec. IV. In particular, the validity of closures based on isotropic functions is discussed.

Although, for the moment, no nonlinear model is proposed, a comparison with experiments made in nonlinear and inhomogeneous turbulent fields seems to prove that our linear results may represent fairly well the effects of rotation on real flows such as encountered in turbomachines. However, to make such a comparison, we must assume, in Sec. V, that the action time of shear and rotation is limited to a value which can be considered as a memory time of turbulence and is directly connected to the presence of nonlinear effects. This remark may provide support to the assumption that rapid distortion can give qualitative results which are valid for more general flows.

II. Correlation and Spectral Equations for a Homogeneous Turbulent Field

The field is assumed to be incompressible and the mean flow is steady in a rotating frame. In this rotating frame, the equation of second-order correlations at two points M and M' is

$$\frac{\partial \overline{u_i u_j'}}{\partial t} + \bar{U}_l \frac{\partial \overline{u_i u_j'}}{\partial X_l} + \bar{U}_l' \frac{\partial \overline{u_i u_j'}}{\partial X_l'} + \left(\frac{\partial \bar{U}_i}{\partial X_l} + 2\epsilon_{ikl}\Omega_k\right)\overline{u_l u_j'}$$
$$+ \left(\frac{\partial \bar{U}_j'}{\partial X_l'} + 2\epsilon_{jkl}\Omega_k\right)\overline{u_l' u_i} + \frac{\partial}{\partial X_l}(\overline{u_i u_l u_j'}) + \frac{\partial}{\partial X_l'}(\overline{u_j' u_l' u_i})$$
$$+ \frac{1}{\rho}\left(\overline{u_j' \frac{\partial p}{\partial X_i}} + \overline{u_i \frac{\partial p'}{\partial X_j'}}\right) - \nu\left(\frac{\partial^2 \overline{u_i u_j'}}{\partial X_k^2} + \frac{\partial^2 \overline{u_i' u_j}}{\partial X_k'^2}\right) = 0 \quad (1)$$

where overbars indicate average values, primes denote quantities at the point M', and Ω_i and ϵ_{ijk} stand for the rate of rotation of the frame and for Ricci's tensor. The terms involving Ω are due to Coriolis forces.

The turbulent field is now assumed to be homogeneous and the mean velocity gradients to be uniform:

$$\lambda_{ij} = \frac{\partial \bar{U}_i}{\partial X_j}$$

Then introducing $\xi_i = X_i' - X_i$ gives

$$\frac{\partial \overline{u_i u_j'}}{\partial t}(\xi,t) + \lambda_{lm}\xi_m \frac{\partial \overline{u_i u_j'}}{\partial \xi_l}(\xi,t) + (\lambda_{il} + 2\epsilon_{ikl}\Omega_k)\overline{u_l u_j'}(\xi,t)$$
$$+ (\lambda_{jl} + 2\epsilon_{jkl}\Omega_k)\overline{u_i u_l'}(\xi,t) - \frac{\partial}{\partial \xi_l}(\overline{u_i u_l u_j'}(\xi,t) - \overline{u_j u_l u_i'}(-\xi,t))$$
$$- \frac{1}{\rho}\left(\frac{\partial \overline{p u_j'}}{\partial \xi_i}(\xi,t) - \frac{\partial \overline{p u_i'}}{\partial \xi_j}(-\xi,t)\right) - 2\nu \frac{\partial^2 \overline{u_i u_j'}}{\partial \xi_k^2}(\xi,t) = 0 \quad (2)$$

or in spectral form:

$$\frac{\partial \Phi_{ij}}{\partial t} - \lambda_{lm} K_l \frac{\partial \Phi_{ij}}{\partial K_m} + (\lambda_{il} + 2\epsilon_{ikl}\Omega_k)\Phi_{lj} + (\lambda_{jl} + 2\epsilon_{jkl}\Omega_k)\Phi_{il}$$
$$- K_l(\theta_{ilj} + \theta_{jli}^*) - (K_i \Sigma_j + K_j \Sigma_i^*) + 2\nu K^2 \Phi_{ij} = 0 \quad (3)$$

where $\Phi_{ij}(\boldsymbol{K})$, $\theta_{mlj}(\boldsymbol{K})$, and $\Sigma_i(\boldsymbol{K})$ are the three-dimensional Fourier transformations of double velocity correlations, triple velocity correlations, and pressure-velocity correlations.

We can now express Σ_i using a Poisson equation and obtain

$$\Sigma_j(\boldsymbol{K},t) = (2\lambda_{lm} + 2\epsilon_{lkm}\Omega_k)\frac{K_l}{K^2}\Phi_{mj} - \frac{K_m K_l}{K^2}\theta_{mlj}(\boldsymbol{K},t) \quad (4)$$

It is interesting to note at this point that the pressure velocity correlation is a sum of three contributions: two linear contributions denoting the action of the mean gradient and the influence of Coriolis forces, and a nonlinear contribution.

Substituting Eq. (4) in Eq. (3) gives

$$\left(\frac{\partial}{\partial t} + 2\nu K^2\right)\Phi_{ij}(K,t) - \lambda_{lm}\left(K_l \frac{\partial \Phi_{ij}}{\partial K_m} + 2\frac{K_i K_l}{K^2}\Phi_{mj}\right.$$
$$\left. + 2\frac{K_j K_l}{K^2}\Phi_{im} - \delta_{il}\Phi_{mj} - \delta_{jl}\Phi_{im}\right) + 2\epsilon_{mkl}\Omega_k(\Phi_{lj}\Delta_{im}$$
$$+ \Phi_{il}\Delta_{jm}) = K_l(\theta_{mlj}\Delta_{im} + \theta_{mlj}^*\Delta_{jm}) \quad (5)$$

with $\Delta_{ij} = \delta_{ij} - (K_i K_j / K^2)$.

We call Γ_{ij} the term due to rotation appearing in Eq. (5):

$$\Gamma_{ij} = 2\epsilon_{mkl}\Omega_k(\Phi_{lj}\Delta_{im} + \Phi_{il}\Delta_{jm}) \quad (6)$$

We now make the hypothesis that triple correlations are negligible. For each allowed $\boldsymbol{K}$, let us introduce three mutually orthogonal unit vectors $\boldsymbol{n}^i(\boldsymbol{K})$, such as $\boldsymbol{n}^1(\boldsymbol{K})$ and $\boldsymbol{n}^2(\boldsymbol{K})$ lie in the plane normal to $\boldsymbol{K}$, while $\boldsymbol{n}^3(\boldsymbol{K})$ is parallel to $\boldsymbol{K}$. In the local frame defined by $\boldsymbol{n}^1$, $\boldsymbol{n}^2$, and $\boldsymbol{n}^3$, calculations can be reduced as Φ_{ij} has only four components and can be expressed as a function of four scalars.[7]

We choose the particular set of $\boldsymbol{n}^i$:

$$\boldsymbol{n}^1(\boldsymbol{K}) = \begin{bmatrix} \dfrac{K_1 K_3}{K\sqrt{K_1^2 + K_1^2}} \\ \dfrac{K_2 K_3}{K\sqrt{K_1^2 + K_2^2}} \\ -\dfrac{\sqrt{K_1^2 + K_2^2}}{K} \end{bmatrix} \qquad \boldsymbol{n}^2(\boldsymbol{K}) = \begin{bmatrix} \dfrac{-K_2}{\sqrt{K_1^2 + K_2^2}} \\ \dfrac{K_1}{\sqrt{K_1^2 + K_2^2}} \\ 0 \end{bmatrix}$$

$$\boldsymbol{n}^3(\boldsymbol{K}) = \begin{bmatrix} \dfrac{K_1}{K} \\ \dfrac{K_2}{K} \\ \dfrac{K_3}{K} \end{bmatrix}$$

and the local frame is then the one introduced by Craya,[7] in which Φ_{ij} can be written

$$\Phi_{ij}'(\boldsymbol{K}) = \begin{bmatrix} N_1 & P + iQ & 0 \\ P - iQ & N_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

In the local frame, Eq. (5) gives an equation which is similar to the one written by Craya, with the additional term due to rotation:

$$\Gamma_{ij}' = 2\omega_{lq}\Omega_l(\epsilon_{iqr}\Phi_{rj}' + \epsilon_{jqr}\Phi_{ir}')$$

in which ω_{ij} is the orthogonal matrix

$$\omega_{ij} = n_j^i(\boldsymbol{K})$$

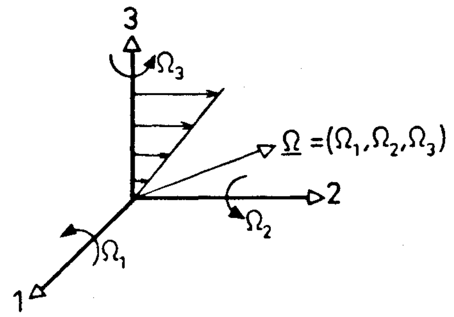

Fig. 1 Uniform shear flow in a rotating frame.

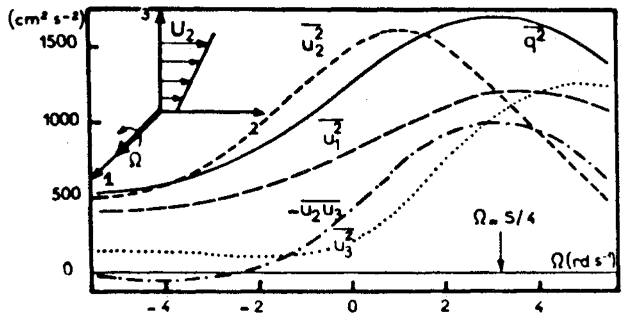

Fig. 2 Components of the Reynolds stress tensor vs rotation of the frame (time $t = 0.25$ s, $St = 3.4$).

We now obtain the set of equations governing the four scalars:

$$\frac{\partial N_1}{\partial t} - K_p \lambda_{pq} \frac{\partial N_1}{\partial K_q} + (2\nu K^2 + 2F_1) N_1 + (2G_1 - 4\omega_{13}\Omega_1) P = 0$$

$$\frac{\partial N_2}{\partial t} - K_p \lambda_{pq} \frac{\partial N_2}{\partial K_q} + (2\nu K^2 + 2F_2) N_2 + (2G_2 + 4\omega_{13}\Omega_1) P = 0$$

$$\frac{\partial P}{\partial t} - K_p \lambda_{pq} \frac{\partial P}{\partial K_q} + (2\nu K^2 + F_1) P + G_1 N_2 + G_2 N_1$$

$$+ 2\omega_{13}\Omega_1 (N_1 - N_2) = 0$$

$$\frac{\partial Q}{\partial t} - K_p \lambda_{pq} \frac{\partial Q}{\partial K_q} + (2\nu K^2 + F_1 + F_2) Q = 0 \tag{7}$$

with

$$\alpha_p = K_p / K, \quad F_1 = \lambda_{pq}\omega_{1p}\omega_{1q}, \quad F_2 = \lambda_{pq}\omega_{2p}\omega_{2q}$$

$$G_1 = \lambda_{pq}\omega_{1p}\omega_{2q} - \lambda_{p1}\alpha_p \frac{\alpha_2\alpha_3}{1-\alpha_3^2} + \lambda_{p2}\alpha_p \frac{\alpha_1\alpha_3}{1-\alpha_3^2}$$

$$G_2 = \lambda_{pq}\omega_{2p}\omega_{1p} + \lambda_{p1}\alpha_p \frac{\alpha_2\alpha_3}{1-\alpha_3^2} - \lambda_{p2}\alpha_p \frac{\alpha_1\alpha_3}{1-\alpha_3^2}$$

Equations (7) admit characteristics which are not modified by the rotating motion of the frame.

Application to Shear Flow

In the case of a mean shear flow (Fig. 1)

$$\lambda_{ij} = S\delta_{i2}\delta_{j3}$$

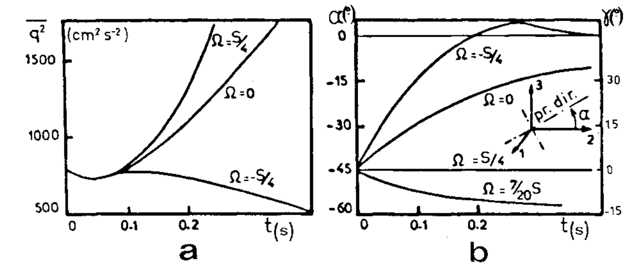

Fig. 3 a) Evolution with time of the turbulent kinetic energy $\overline{q^2}$, for different values of Ω. b) Evolution with time of the angle α between the principal directions of the Reynolds stress and the axes of the rotating frame, and of the angle γ between the principal directions of the Reynolds stress tensor and the principal axes of $\partial \bar{U}_i/\partial X_j + \partial \bar{U}_j/\partial X_i$.

Equations (7) give

$$\frac{\partial N_1}{\partial t} - SK_2 \frac{\partial N_1}{\partial K_3} + \left(2\nu K^2 - 2S\frac{K_2K_3}{K^2}\right) N_1$$

$$-4\left(\Omega_1 \frac{K_1}{K} + \Omega_2 \frac{K_2}{K} + \Omega_3 \frac{K_3}{K}\right) P = 0$$

$$\frac{\partial N_2}{\partial t} - SK_2 \frac{\partial N_2}{\partial K_3} + 2\nu K^2 N_2 + 4\left[\left(\Omega_1 - \frac{S}{2}\right)\frac{K_1}{K}\right.$$

$$\left. + \Omega_2 \frac{K_2}{K} + \Omega_3 \frac{K_3}{K}\right] P = 0$$

$$\frac{\partial P}{\partial t} - SK_2 \frac{\partial P}{\partial K_3} + \left(2\nu K^2 - S\frac{K_2K_3}{K^2}\right) P + 2\left[\left(\Omega_1 - \frac{S}{2}\right)\frac{K_1}{K}\right.$$

$$\left. + \Omega_2 \frac{K_2}{K} + \Omega_3 \frac{K_3}{K}\right] N_1 - 2\left(\Omega_1 \frac{K_1}{K} + \Omega_2 \frac{K_2}{K} + \Omega_3 \frac{K_3}{K}\right) N_2 = 0$$

$$\frac{\partial Q}{\partial t} - SK_2 \frac{\partial Q}{\partial K_3} + \left(2\nu K^2 - S\frac{K_2K_3}{K^2}\right) Q = 0 \tag{8}$$

III. Results of the Linear Spectral Computation

A numerical approach is used to solve the set of Eqs. (8). All the computations presented here are made using isotropic initial conditions. The initial spectrum corresponds to the experiment of Comte-Bellot and Corrsin.[8] Only results concerning the case of a rotation about the first axis (Fig. 1) are given here, as it is the most interesting case. The value of the shear is 13.6 s^{-1}, which corresponds to the experiment of Rose,[9] and the computation of Loiseau,[10] in steady frame.

The curves of Fig. 2 give the variation of $\overline{q^2}$, $\overline{u_1^2}$, $\overline{u_2^2}$, $\overline{u_3^2}$, and $\overline{u_2u_3}$ with Ω, at a time $t = 0.25$ s, which corresponds to a value of the nondimensional product St of 3.4. Stabilizing effects appear for negative values of Ω/S, and destabilizing effects for positive values of Ω/S. These effects are particularly strong on the Reynolds stress $\overline{u_2u_3}$.

Figure 3 shows the evolution with time of our results. In the first stage, rotation does not affect the turbulent kinetic energy. It only alters the orientation of the principal axes of the Reynolds stress tensor: it is known that Coriolis forces are not producing energy, but are responsible for a redistribution between components. Later, the turbulent kinetic energy is affected, as the change in the orientation alters the production term in the equation governing $\overline{q^2}$. The fluctuating motion can then extract a different amount of energy from the mean flow.

In order to make a more complete presentation of our results, a visualization of the three-dimensional spectral tensor $\Phi_{ij}(K)$ is proposed. On a sphere of radius K, the

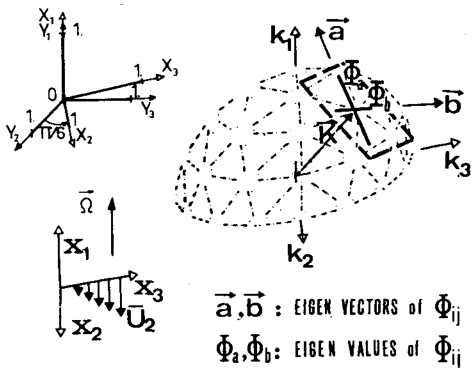

Fig. 4 Visualization of the tensor $\Phi_{ij}(K)$ over spheres of radius K. $(0, X_1, X_2, X_3)$ is the frame used to describe the turbulent motion, $(0, Y_1, Y_2, Y_3)$ is the frame in which perspectives are drawn, and $(0, K_1, K_2, K_3)$ is the equivalent of $(0, X_1, X_2, X_3)$ in spectral space.

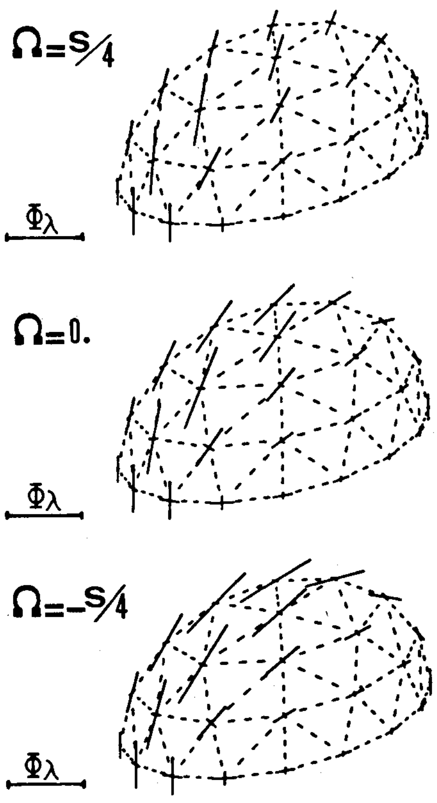

Fig. 5 Visualization of the tensor $\Phi_{ij}(K)$ over a sphere of radius K, (same conventions as in Fig. 5), for $\Omega=-S/4$, $\Omega=0$, and $\Omega=+S/4$; $St=1.36$ and $K=0.5$ cm^{-1}. Scale in the Y_1Y_3 plane: $\Phi_\lambda=4.166$ 10^{-8} m^{5}s^{-2}.

principal axes of $\Phi_{ij}(K)$ are visualized, and two segments proportional to the two eigenvalues are plotted. The spheres, as well as the segments, are plotted in perspective in the $\{Y_1, Y_2, Y_3\}$ frame defined in Fig. 4. It is of interest to note, on the visualization in Fig. 5, that the effects of Coriolis forces are mainly to induce a rotation of Φ_{ij} about the K axis, for given values of K. The distribution of energy on the sphere is not much modified by the presence of rotation.

Before examining results concerning the behavior of the pressure-velocity correlation, we must remark that for homogeneous turbulence, the quantity

$$\overline{u_i \frac{\partial p}{\partial X_j} + u_j \frac{\partial p}{\partial X_i}}$$

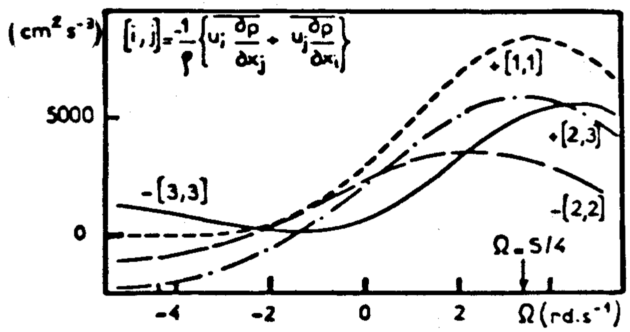

Fig. 6 Velocity-pressure correlations vs Ω, for $St=3.4$.

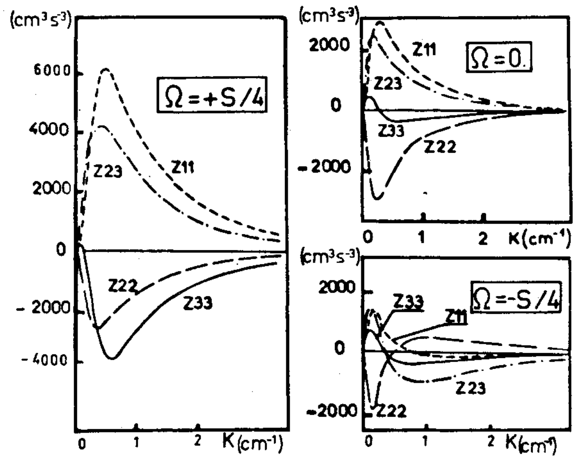

Fig. 7 Spectra of velocity-pressure correlation for $\Omega=+S/4$, $\Omega=0$, and $\Omega=+S/4$, for $St=3.4$.

can also be written

$$\overline{-p\left(\frac{\partial u_i}{\partial X_j}+\frac{\partial u_j}{\partial X_i}\right)}$$

and that furthermore, the two quantities have the same spectrum. Therefore we will not distinguish the two quantities.

The curves in Fig. 6 show that velocity-pressure correlations are strongly influenced by the rotating motion of the frame. Spectra of those correlations, plotted in Fig. 7, exhibit changes of sign when Ω/S is negative, which means that the role played by pressure is not the same whether big or small eddies are considered. This "pathological" behavior occurs when the rotation of the frame adds to the mean rotational value associated with the shear (stabilizing case).

In return, when the rotating motion of the frame tends to compensate the effects of the rotation induced by the shear, a more common behavior of pressure is found. For example, for the value $\Omega=S/4$ (Fig. 7), the spectra of the pressure velocity correlation are similar to those obtained in plane strain, which can be related to the fact that, in both cases, the principal axes of $\overline{u_iu_j}$ and of the mean gradient are aligned ($\gamma=0$ in Fig. 3b). Furthermore, the "usual" role of pressure, to redistribute the energy production between components of the Reynolds stress tensor, appears to be clear only when Ω/S is positive (γ small).

Therefore, as a general trend, we can say that complex behaviors of pressure terms occur when Ω takes values far from $S/4$, i.e., when the angle γ between the principal axes of the mean gradients and of $\overline{u_iu_j}$ increases. The case of the shear in steady frame then appears as a particular case in which a "pathological" effect already exists on the Z_{33} component (Fig. 7), owing to the rotational part of the shear.

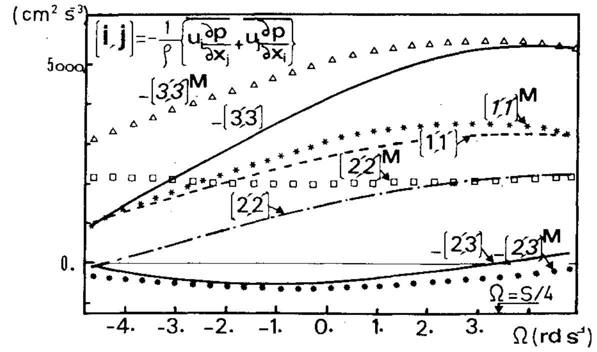

Fig. 8 Velocity-pressure correlations $[i,j]$, and their modeled forms (rapid part) $[i,j]^M$, vs Ω, for $St = 1.36$. Both tensors are expressed in the principal axes of $\partial \bar{U}_i/\partial X_j + \partial \bar{U}_j/\partial X_i$.

It is interesting to note that similar "pathological" spectra are obtained in the case of a turbulence subjected to both shear and buoyancy effects, when gravity has a stabilizing action.[11]

IV. Application to Turbulence Modeling

Generalities About Turbulence Modeling

Since the nonlinear transfer of energy between eddies of various sizes and the effects due to inhomogeneity are not taken into account by rapid distortion, more complete models are needed to represent complex flows.

As cumbersome computations would result from introducing nonlinear terms directly deduced from the analytical theories of turbulence[12] in the equation governing $\Phi_{ij}(K)$ and from solving this equation at each point of an inhomogeneous turbulent field, predictions of complex flows are usually done by using a more simple set of equations. For example, the equations of double correlations at one point are often chosen.[13,14]

Computations are then reduced, but in return the pressure-velocity correlation becomes an unknown quantity which has to be modeled by using a closure relation. Before being modeled, this term is generally split into two parts: a linear contribution, which is called the "rapid" part of pressure, and a nonlinear contribution, which is called the return-to-isotropy part. It is usually believed that the action of triple correlations can be sufficiently separated from the linear process to allow the modeling of each part of pressure separately. The rapid part is then modeled with reference to a situation in which only linear mechanisms are present. Moreover this situation is generally taken as homogeneous. Therefore the results of two-point linear studies are used as reference to test this modeling of the rapid part of pressure.[13]

Such a test has already been made in Galilean frames, and we know that the classical modelings[13,14] are in good agreement with linear spectral results in the case of a pure strain deformation. The confrontation is less satisfactory in the case of a shear flow, but is still acceptable.

In the next paragraph, we shall use the results of Sec. III to extend the comparison to rotating frames. We will also make the same kind of comparison with another model, proposed by Cambon,[15] and preserving spectral information.

Confrontation Between Classical Modelings and Linear Spectral Results: Discussion

For given values of t, we compare the values of the component of the tensor

$$\frac{1}{\rho}\left(\overline{u_i\frac{\partial p}{\partial X_j}} + \overline{u_j\frac{\partial p}{\partial X_i}}\right)_{\text{linear}}$$

obtained in rapid distortion, to the values resulting from reporting the corresponding $\overline{u_iu_j}$ in the modeling relations proposed by Launder[14] and Lumley.[13] An example of comparison is given in Fig. 8, for $St = 1.36$ and for $C = 0.2456$ in Lumley's model.

As a general result, we can say that a rotation of the frame corresponding to $\Omega/S > 0$ tends to improve the validity of the modeled pressure. In return, negative values of Ω/S tend to introduce errors in the models. It is relevant to note that good results are obtained when the rotating motion of the frame tends to compensate the rotation induced by the shear, as bad results occur when both rotations have the same sign. Therefore the bad behavior of the models seems to be due to the presence of rotation. We shall now try to explain this behavior.

We must first recall how modeling relations are constructed. The linear part of pressure is written

$$\frac{1}{\rho}\left(\overline{u_i\frac{\partial p}{\partial X_j} + u_j\frac{\partial p}{\partial X_i}}\right) = 2\left(\frac{\partial \bar{U}_l}{\partial X_m} + \epsilon_{lkm}\Omega_k\right) \times \int_K \frac{K_iK_l}{K^2}\Phi_{mj}(K) + \frac{K_jK_l}{K^2}\Phi_{im}(K)\mathrm{d}K \tag{9}$$

the integral in Eq. (9), which we call a_{ijlm}, is then expressed as an isotropic function of anisotropic arguments. Usually, it is written

$$a_{ijlm}(t) = F\left[b_{nk}(t), \overline{q^2}(t)\right]$$

where F is a unknown isotropic function, t is given, and b_{ij} is the deviatoric part of $\overline{u_iu_j}$

$$b_{ij} = \overline{u_iu_j}/\overline{q^2} - 2\delta_{ij}/3$$

Under the assumption that F is linear, only one constant appears in the final form of a_{ijlm}.

To understand the action of Coriolis forces on the behavior of the modeled forms, we must now remark that a rotation of the Reynolds stress tensor will result in the same rotation for the modeled a_{ijlm}. It is a consequence of the isotropy of F. Therefore Coriolis forces, which are inducing a rotation of $\overline{u_nu_k}$ about the Ω axis, will also induce a rotation of the modeled a_{ijlm} tensor. Let us now consider the action of Coriolis forces on the exact form of a_{ijlm}. Figure 5 shows that Coriolis forces induce a rotation of Φ_{ij}, for a given K, but do not significantly alter the distribution of Φ_{ij} over spheres of radius K. Therefore the exact a_{ijlm}, which is an integral of terms taking into account the distribution of Φ_{ij} in K space, cannot be simply deduced from the case $\Omega = 0$ by a single rotation. This discrepancy between the behavior of the modeled form and the behavior of the exact tensor explains why the rotation of the frame, and even the rotational part of the shear, alters the validity of closures based on isotropic functions.

A more enlightening illustration of the inadequacy of isotropic modeling in rotating flows can be given if we consider the model proposed by Cambon,[15] which is also based on isotropic functions.

In order to preserve spectral information, Cambon suggested using the equations governing the integrals of $\Phi_{ij}(K)$ over spheres of radius K:

$$\varphi_{ij}(|K|) = \int_\Sigma \Phi_{ij}(K)\mathrm{d}\Sigma(K)$$

The problem is then to model the pressure term

$$a'_{ijlm}(|K|) = \int_\Sigma \frac{K_iK_l}{K^2}\Phi_{mj}(K) + \frac{K_jK_l}{K^2}\Phi_{im}(K)\mathrm{d}\Sigma(K) \tag{10}$$

which is an integral over a sphere of radius K.

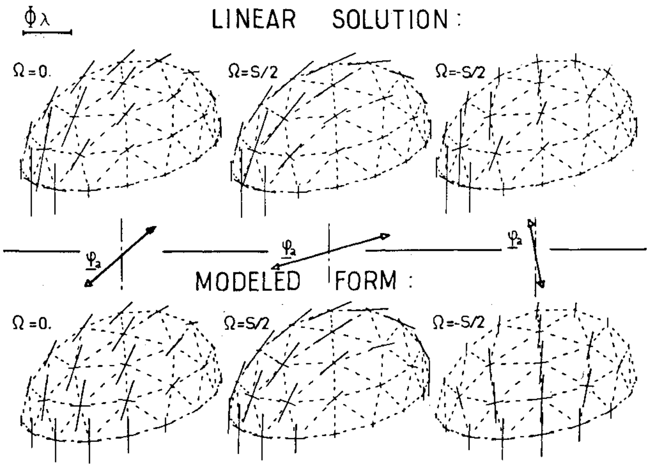

Fig. 9 Comparison between linear solution $\Phi_{ij}(K)$ and its modeled form $\Phi^M_{ij}(K)$, $t=0.08$ s ($St=1.088$), $K=1$ cm^{-1}. Same conventions as in Fig. 5. Scale in the $Y_1 Y_3$ plane: $\Phi_\lambda = 5.10^{-9}$ m^5 s^{-2}. φ_a shows the principal direction of φ_{ij}.

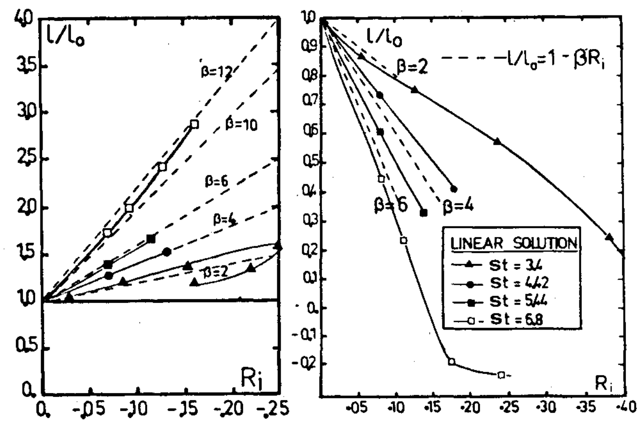

Fig. 10 Mixing length ratio vs Richardson number. Comparison between linear computation and the relation used in turbomachinery. (l is affected by the sign of $-\overline{u_2 u_3}$).

As Cambon pointed out, the same expression of a'_{ijlm} will be obtained by writing

$$a'_{ijlm}(|K|,t) = F[H_{nk}(|K|,t), E(|K|,t)]$$

or by modeling $\Phi_{ij}(K)$ directly,

$$\Phi^M_{ij}(K,t) = G\left[H_{nk}(|K|,t), E(|K|,t), \frac{K_l}{K}\right]$$

and then replacing in the integral (10), F and G being isotropic and linear, and with

$$H_{ij} = \frac{\varphi_{ij}}{2E} - \frac{\delta_{ij}}{3}, \quad E = \frac{\varphi_{ii}}{2}$$

We compare, by visualizing, the two tensors $\Phi_{ij}(K)$ and $\Phi^M_{ij}(K)$. Figure 9 shows the tendency of the distribution of Φ^M_{ij} in K space to follow the principal axes of φ_{ij} when they are rotating, whereas the distribution of the exact from Φ_{ij} does not change.

We can then consider that the use of isotropic functions is not fitting because of the orientation of the turbulent field induced by the rotating motion of the frame. Therefore the modeling of linear terms could probably be greatly improved by the use of a function satisfying only axial properties, or by using isotropic functions having the direction of $\boldsymbol{\Omega}$ as argument, which is equivalent. We must at this point, mention the attempt of Lumley and Khajeh-Nouri[16] who

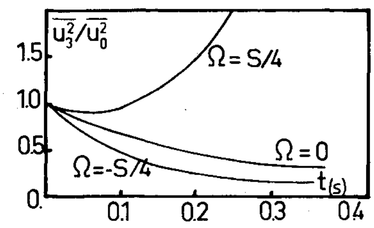

Fig. 11 Evolution with time of $\overline{u_3^2}$.

included dependence on the vorticity tensor $\Omega_{ij} = (\partial \overline{U}_i / \partial X_j) - (\partial \overline{U}_j / \partial X_i)$, and another model due to Lumley[14] which takes the whole deformation tensor into account. Both models have been tested and none gives better agreements with our results than the simplest isotropic one. However, no constant optimizing has been performed and it is our opinion that improvements could be made in that way.

V. Limitations of the Rapid Distortion Model and Application to Turbomachinery

When rotating centrifugal impellers are studied, the effects of rotation on turbulent boundary layers are usually taken into account by a relation for the mixing-length ratio l/l_0[6,17,18]:

$$l/l_0 = 1 - \beta Ri \tag{11}$$

in which l is defined by

$$l = (|\overline{u_2 u_3}|)^{1/2}/S$$

l_0 is the same quantity without rotation and Ri is the local Richardson number:

$$Ri = [-2\Omega(S-2\Omega)]/S^2$$

The values of β are based on experiments.

We have plotted in Fig. 10 the values of the ratio l/l_0 deduced from our linear spectral study. For quite a big range of Ri, the linearity of l/l_0 as a function of Ri is nearly obtained, which provides support to the mixing-length relation.[11] β is found to be an increasing function of St.

These are the results of rapid distortion. They can rigorously apply to a real machine only if the residence time of turbulence inside the blade-to-blade channel is short when compared to a characteristic time of turbulence (and if inhomogeneity can be neglected). Whether they can provide information concerning more general flows, involving nonlinear effects, will now be discussed.

The first symptom of the pathology resulting from the neglect of triple correlations appears on the evolution of the turbulent kinetic energy $\overline{q^2}$ (Fig. 3). It is known that when time becomes too long, $\overline{q^2}$ takes unphysical values in rapid distortion. However, if relative dimensionless ratios, like $\overline{u_i u_j}(t)/\overline{q^2}(t)$, are considered, it is generally believed that the results of the linear approach remain valuable for larger values of t. This is particularly true concerning dimensionless quantities related to small wavenumbers and has been verified by experiments for steady frame flows. Later, pathological behavior appears on the relative quantities. In particular, in the case of shear flows, Deissler[19] has pointed out the overestimated decay of the ratio $\overline{u_3^2}/\overline{q^2}$.

In the present case, we see no reason to think that the presence of Coriolis forces will greatly affect the validity of rapid distortion, since the decay of $\overline{u_3^2}$ remains of the same order as it is in steady frame even for stabilizing cases (Fig. 11). Therefore we think that our results, as far as relative quantities are concerned, will have a range of validity larger than the one corresponding to the strict hypothesis used in linear calculations.

Support for this assumption may be provided by a comparison with large eddy simulation. Shaanan et al.[20] (Fig. 12) have found $\beta \sim 1.7$ for $St \sim 3.17$, as we find $\beta \sim 2$ for $St = 3.4$, meanwhile, for such values of St, nonlinear transfer is not negligible in a situation corresponding to the one simulated.

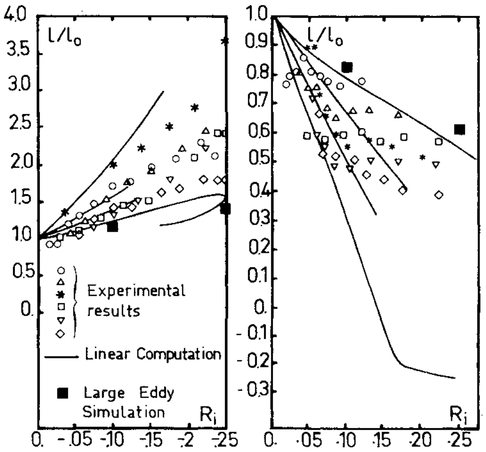

Fig. 12 Mixing length ratio vs Richardson number. Comparison between linear spectral computation, measurements from Johnston[4] and large eddy simulation from Shaanan.[20]

In order to know what happens for even higher values of t, and inhomogeneous flows, a comparison between our results and measurements made by Johnston[4] is proposed in Fig. 12. Values of β corresponding to the whole range of the experimental results are found by our linear computation for values of St between 3.4 and 6.8. An estimation of the values of St corresponding to the experiments, t being the residence time of turbulence in the rotating channel, gives larger values of St. Comparable St can only be found if we limit t to a value which can be considered as a memory time of turbulence: a turnover time connected to nonlinear and inhomogeneous effects. It then seems that the linear spectral computation could provide a way of estimating the Coriolis effects in turbomachines, if a characteristic time of nonlinear processes is known.

An experimental study has also been made. A description of the testing impeller and results are presented in other papers.[21,22] It is interesting to note that Coriolis effects are found to be in qualitative agreement with the linear theory in the machine which is much more similar to a real centrifugal compressor than the installations used in previous experiments.[4,5]

VI. Conclusion

Important characteristics of turbulent flows in rotating frames have been found by our linear computation. We have seen that qualitative and even quantitative results are obtained which could be used for turbomachinery studies. At the moment, the lack of nonlinear and inhomogeneous terms in the computation does not permit further comparison with experiments.

Our results emphasize the complex role played by the pressure-velocity correlation in rotating frames. In particular, the need of being careful when modeling the rapid part of pressure has been pointed out.

The case of the action of Coriolis forces on turbulence can be considered as being an example of turbulent flows subjected to external forces. The force could be a magnetic field in magneto-hydro-dynamic turbulence, or gravity in a problem with buoyancy. In both cases, as well as in the case of rotation, the use of isotropic functions for the modeling may be inadequate.

In a following paper[23] a closure based on eddy damped quasi-normal assumptions will be introduced to take into account nonlinear effects in the equations of the three-dimensional spectrum $\Phi_{ij}(K)$. It is hoped that the resulting model will be particularly suitable for predicting anisotropic homogeneous turbulent fields submitted to external forces, since the linear effects will be exactly taken into account.

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